THH of the connective cover of $L_{K(1)}S$

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1. Waldhausen's program
2. topological Hochschild homology
3. THH-May spectral sequence
4. THH of the connective cover of $L_{K(1)}S$

Chromatic localizations

One of the most fundamental problems in algebraic topology is to understand the homotopy groups of spheres. The chromatic perspective tells us we can approach this via localizations

$$S_{(p)} \longrightarrow \ldots \longrightarrow L_nS \longrightarrow L_{n-1}S \longrightarrow \ldots \longrightarrow L_1S \longrightarrow L_0S$$

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where p is some fixed prime, and $L_n := L_{K(n)\vee\ldots\vee K(1)\vee K(0)}$. These are localizations in the sense of Bousfield which are taken at wedges of the Morava K-theory spectra $K(n)$. The spectra $K(n)$ along with $H\mathbb{Q}$ and $H\mathbb{F}_p$ can be considered the "prime fields" in the category ring spectra in a way made precise by Devinatz-Hopkins-Smith.

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Waldhausen proposed that one should compute the algebraic K-theory of these connective covers of the localizations of the sphere. Algebraic K-theory is notoriously difficult to compute, but one way that has been successful is to use the trace map from algebraic K-theory to topological Hochschild homology. We therefore approach this problem by computing $THH(\ell_1(S))$ (after smashing with a Smith-Toda complex).

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Once TC is known, there are methods for going from TC to algebraic K-theory.

Thm:(Dundas, Goodwillie, McCarthy)

Let A be an S-algebra with a surjective ring map $\hat{\mathbb{Z}}_p \longrightarrow \pi_0(A)$. Then the cyclotomic trace fits in a cofiber sequence

$$\hat{\mathcal{K}}(A)_p \longrightarrow \hat{TC}(A)_p \longrightarrow \Sigma^{-1} H\hat{\mathbb{Z}}_p$$

Trace Methods

To compute topological cyclic homology, we use the fact that $THH(A)$ is a cyclotomic S^1-equivariant spectrum. We can therefore define Frobenius and restriction maps,

$$THH(A)^{C_{p^n}} \underset{R}{\overset{F}{\rightrightarrows}} THH(A)^{C_{p^{n-1}}}$$

We define $TC(A;p) := holim_{R,F} THH(A)^{C_{p^n}}$ and $TC(A)_p \simeq TC(A;p)_p$, so this is enough for computations up to p-completion.

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$$d_i = 1 \wedge 1 \wedge ... \wedge \mu \wedge ... \wedge 1 \text{ for } 0 \leq i < n,$$

$$d_n = (\mu \wedge 1 \wedge ... \wedge 1) \circ \tau_n,$$

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Topological Hochschild homology

$$\mathrm{THH}(R) \simeq |\mathrm{Bar}_{cyc}(R)|$$

where $| - |$ denotes geometric realization of a simplicial ring spectrum.

May-type spectral sequence for THH

Thm: (Salch)

Given a filtration $I_\bullet$ of a commutative ring spectrum R, there exists a spectral sequence $E_1^{*,*} \cong \pi_{*,*}(THH(E_0(I_\bullet))) \Rightarrow \pi_*(THH(R))$.

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Def:

A filtration of a commutative ring spectrum R, is a sequence of cofibrations

$$\ldots \longrightarrow I_n \longrightarrow I_{n-1} \longrightarrow \ldots \longrightarrow I_1 \longrightarrow I_0 = R$$

with structure maps

$$\rho_{i,j} : I_i \wedge I_j \longrightarrow I_{i+j}$$

satisfying compatibility, commutativity, associativity, and unitality.

Application

Due to Devinatz-Hopkins, we have a fiber sequence

$$L_{K(1)}S \longrightarrow E_1 \longrightarrow E_1$$

where $L_{K(1)}S = E_1^{h\mathbb{G}_1}$ where $\mathbb{G}_1 \cong \mathbb{Z}_p^\times$ is the height 1 Morava stabilizer group.

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$$\ell_{K(1)}S \longrightarrow \ell_p \xrightarrow{1-\psi_q} \Sigma^{2p-2}\ell_p$$

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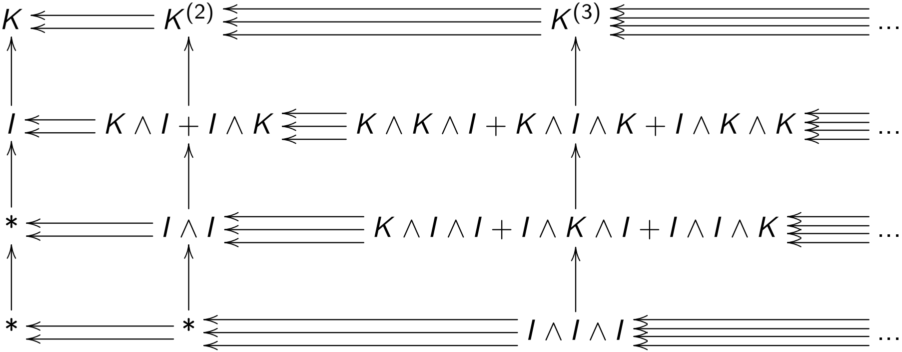
Thm: (A-K and Salch)

The sequence of cofibrations $* \longrightarrow \Sigma^{2p-3}\ell_p \longrightarrow \ell_{K(1)}S$ is a filtration of the commutative ring spectrum $\ell_{K(1)}S$.

Construction of the spectral sequence

We have a filtration of the simplicial ring spectrum whose geometric realization is $\mathrm{THH}(\ell_{K(1)}S)$. We write $K := \ell_{K(1)}S$ and $I = \Sigma^{2p-3}\ell_p$, then there is a filtration of $\mathrm{THH}(K)$ (where "+" indicates a colimit of an appropriate diagram):



Construction of the spectral sequence

Letting M_0 be the first row, M_1 the second row, and so on. We get a tower of cofibrations

$$\begin{array}{ccc}
\vdots & & \\
\downarrow & & \\
|M_2| & \longrightarrow & |M_2|/|M_3| \\
\downarrow & & \\
|M_1| & \longrightarrow & |M_1|/|M_2| \\
\downarrow & & \\
|M_0| & \longrightarrow & |M_0|/|M_1|
\end{array}$$

The spectral sequence is the spectral sequence of the exact couple associated to this tower of cofibrations after applying any homology theory including stable homotopy.

topological Hochschild homology of $\ell_{K(1)}S$

The input of the spectral sequence is $E_1^{*,*} = \pi_{*,*}(THH(\hat{\ell}_p \ltimes \Sigma^{2p-3}\hat{\ell}_p))$, where $\hat{\ell}_p \ltimes \Sigma^{2p-3}\hat{\ell}_p$ is the trivial square-zero extension of $\hat{\ell}_p$ by the $\hat{\ell}_p$-bimodule $\Sigma^{2p-3}\hat{\ell}_p$.

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Lemma: (Schwanzl-Vogt-Waldhausen)

For K a commutative S algebra and R, A commutative K-algebras

$$\mathrm{THH}^K(R \wedge_K A) \simeq \mathrm{THH}^K(R) \wedge_K \mathrm{THH}^K(A)$$

Bökstedt spectral sequence

There is a spectral sequence for R a commutative S-algebra

$$E_2^{*,*} = HH_*(H_*(R,k)) \Rightarrow H_*(THH(R),k).$$

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In the case $R = S \ltimes \Sigma^{2p-3} S$, we know $H_*(R,k) = E(x)$ with $|x| = 2p-3$. By Cartan-Eilenberg's change of rings argument (See McClure-Staffeldt), we get that $HH_*(E(x)) = E(x) \otimes \Gamma(\sigma x)$. Where $\Gamma(\sigma x)$ is a divided power algbera.

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Splitting

There is an additive splitting, $|THH(R)| \simeq |C_0| \vee |C_1| \vee \ldots$, where C_i is given by the i-th row below.

$$S \leftleftarrows S \begin{smallmatrix}\leftarrow\\ \leftarrow\\ \leftarrow\end{smallmatrix} S \begin{smallmatrix}\leftarrow\\ \leftarrow\\ \leftarrow\\ \leftarrow\end{smallmatrix} \ldots$$

$$\Sigma^{2p-3}S \leftleftarrows (\Sigma^{2p-3}S)^{\vee 2} \begin{smallmatrix}\leftarrow\\ \leftarrow\\ \leftarrow\end{smallmatrix} (\Sigma^{2p-3}S)^{\vee 3} \begin{smallmatrix}\leftarrow\\ \leftarrow\\ \leftarrow\\ \leftarrow\end{smallmatrix} \ldots$$

$$* \leftleftarrows \Sigma^{4p-6}S \begin{smallmatrix}\leftarrow\\ \leftarrow\\ \leftarrow\end{smallmatrix} (\Sigma^{4p-6}S)^{\vee 3} \begin{smallmatrix}\leftarrow\\ \leftarrow\\ \leftarrow\\ \leftarrow\end{smallmatrix} \ldots$$

$$* \leftleftarrows * \begin{smallmatrix}\leftarrow\\ \leftarrow\\ \leftarrow\end{smallmatrix} \Sigma^{6p-9}S \begin{smallmatrix}\leftarrow\\ \leftarrow\\ \leftarrow\\ \leftarrow\end{smallmatrix} \ldots$$

$THH(S \ltimes \Sigma^{2p-3}S)$

It is clear that $|C_0| = S$, and for $n \geq 1$ we see for dimension reasons:

$$\tilde{H}_i(|C_n|, k) \cong \begin{cases} \Sigma^i k\{x^{\otimes n}\} \text{ if } i = n(2p-2)-1 \\ \Sigma^i k\{1 \otimes x^{\otimes n}\} \text{ if } i = n(2p-2) \\ 0 \text{ otherwise} \end{cases}$$

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Thus, $|C_n| \simeq \Sigma^{n(2p-2)-1}S \vee \Sigma^{n(2p-2)}S$. We get an additive splitting:

$$\mathrm{THH}(S \ltimes \Sigma^{2p-3}S) \simeq S \vee \bigvee_{i \geq 1}(\Sigma^{(2p-2)i-1}S \vee \Sigma^{(2p-2)i}S)$$

Then applying the lemma of Schwanzl-Vogt-Waldhausen, we get

$$\begin{aligned} &THH(\ell \ltimes \Sigma^{2p-3}\ell) \simeq \\ &THH(\ell) \wedge (S \vee \textstyle\bigvee_{i \geq 1}(\Sigma^{(2p-2)i-1}S \vee \Sigma^{(2p-2)i}S)) \simeq \\ &THH(\ell) \vee \textstyle\bigvee_{i \geq 1}(\Sigma^{(2p-2)i-1}THH(\ell) \vee \Sigma^{(2p-2)i}THH(\ell)) \end{aligned}$$

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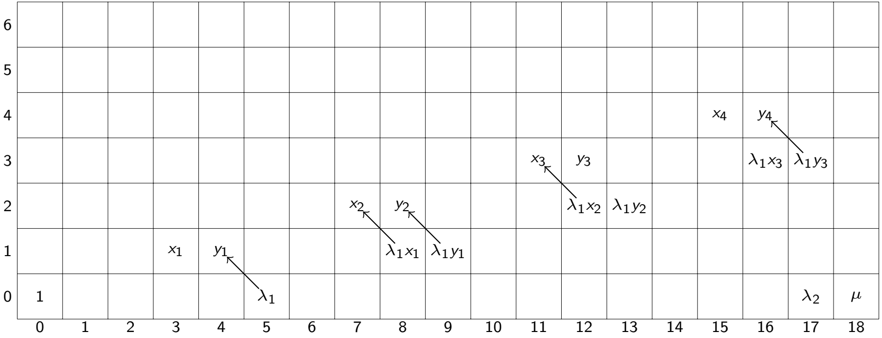
Due to McClure-Staffeldt, $V(1)_*(THH(\ell)) \cong E(\lambda_1, \lambda_2) \otimes P(\mu)$, so the E_2 page of the THH-May spectral sequence in V(1)-homotopy is as follows.

$V(1)$-smash THH-May SS at $p = 3$



Thee element $x_1 = x \otimes 1$, $x_i = x \otimes \gamma_i(\sigma x)$ for $i > 1$, and $y_i = 1 \otimes \gamma_i(\sigma x)$. This a picture of

$$E_2 \cong E(\lambda_1, \lambda_2) \otimes P(\mu) \otimes E(x) \otimes \Gamma(\sigma x)$$

where $|\lambda_1| = (2p-1, 0)$, $|\lambda_2| = (2p^2 - 1, 0)$, $|\mu| = 2p^2$ and $|x| = 2p - 3$ and $|\sigma x| = 2p - 2$.

Differentials

The connective cover of the $K(1)$-local sphere is known to be homotopy equivalent to the p-complete connective image of J. The Homology with $\mathbb{F}_p$ coefficients of $\text{THH}(\hat{j}_p)$ is known due to Angeltveit-Rognes.

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Differentials

Using a version of the THH-May spectral sequence with coefficients, we can show that there is a cofiber sequence

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Thm: (A-K and Salch)

$V(1)_*(THH(\ell_{K(1)}S)) \cong E(x, \lambda_1(\gamma_{p-1}(\sigma x)), \lambda_2) \otimes \Gamma(\gamma_p(\sigma x)) \otimes P(\mu)$

Redshift?

The connective cover the K(1)-local sphere is a chromatic height one spectrum. We would therefore, like to consider the effects on chromatic level of taking algebraic K-theory. To do this, $V(1)$ homotopy is the right thing to consider.

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Red shift conjecture

For B a height one spectrum, the map

$$V(1) \wedge K(B) \longrightarrow v_2^{-1} V(1) \wedge K(B)$$

is an isomorphism in homotopy in sufficiently high degrees.

This would mean that the telescopic complexity of K(B) is 2.

Thank You

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