

Homotopie et Homologie

Exercise Set 5

17.10.2013

1. Let (X, x_0, μ) be an H -space. Show that if $f : (X, x_0) \rightarrow (X', x'_0)$ is a based homotopy equivalence, then X' admits an H -space structure with respect to which f is an H -map. Show that, moreover, if X is actually an H -group, then so is X' . In other words, “to be an H -space (respectively, H -group)” is a homotopy invariant notion.
2. Prove that any H -map of H -groups preserves homotopy inversion up to homotopy, i.e., if (X, x_0, μ, σ) and $(X', x'_0, \mu', \sigma')$ are H -groups and $f : (X, x_0, \mu) \rightarrow (X', x'_0, \mu')$ is an H -map, then $f\sigma \simeq_* \sigma'f$.
3. Let (X, x_0, μ) and (X', x'_0, μ') be H -spaces. Prove that the cartesian product $X \times X'$ admits an H -multiplication μ''
 - (a) with respect to which the projection maps from $X \times X' \rightarrow X$ and $X \times X' \rightarrow X'$ are H -maps, and
 - (b) such that the unique continuous map $(f, f') : Y \rightarrow X \times X'$ induced by a pair of H -maps $f : (Y, y_0, \nu) \rightarrow (X, x_0, \mu)$ and $f' : (Y, y_0, \nu) \rightarrow (X', x'_0, \mu')$ is also an H -map.
4. Let (X, x_0, μ) be an H -space, and let (Y, y_0) be any based space. Show that $\text{Map}((Y, y_0), (X, x_0))$ admits an H -space structure such that for every based, continuous map $f : (Y, y_0) \rightarrow (Z, z_0)$, the induced map

$$f^* : \text{Map}((Z, z_0), (X, x_0)) \rightarrow \text{Map}((Y, y_0), (X, x_0))$$

is an H -morphism.