Homotopie et Homologie
Exercise Set 12

12.12.2013

1. *Complex projective space* of dimenion n is

$$\mathbb{C}P^n = \mathbb{C}^{n+1} \setminus \{0\}/\sim$$

where $z \sim z'$ if and only if there exists $\lambda \in \mathbb{C}$ such that $z = \lambda z'$. Prove that $SP^n(S^2) \cong \mathbb{C}P^n$.

2. (Eilenberg-MacLane spaces)

Definition 1. Let G be an abelian group, and let $n \geq 2$. An *Eilenberg-MacLane space of type* (G, n) is a pointed space (X, x_0) such that $\pi_n(X, x_0) \cong G$ and $\pi_k(X, x_0) = 0$ if $k \neq n$.

Conventions 2. An Eilenberg-MacLane space of type (G, n) is usually denoted $K(G, n)$.

(a) Show that $SP(S^n)$ is an Eilenberg-MacLane space of type $(\mathbb{Z}, n)$ for all $n \geq 1$.

(b) Let $m \in \mathbb{N}$, and define $\lambda_m : S^1 \to S^1$ by $\lambda_m(e^{i\theta}) = e^{im\theta}$. Prove that $\pi_1\lambda_m : \pi_1 S^1 \to \pi_1 S^1$ is given by multiplication by m.

(c) Prove that $SP(C_{\lambda_m})$ is an Eilenberg-MacLane space of type $(\mathbb{Z}/m, 1)$.
Hint 3. Use the quasi-fibration obtained by applying SP to the sequence $S^1 \xrightarrow{\lambda_m} S^1 \to C_{\lambda_m}$.

(d) Let $n > 1$. Prove that $SP(\Sigma^{n-1}C_{\lambda_m})$ is an Eilenberg-MacLane space of type $(\mathbb{Z}/m, n)$.

(e) Let G be any finitely generated abelian group, and let $n \geq 1$. Explain how to construct an Eilenberg-MacLane space of type $K(G, n)$.