Homotopical algebra
Exercise Set 8

16.04.2018

Exercise 3 is to be handed in on 23.04.2018.

1. Let $(\mathsf{M}, \mathsf{WE}, \mathsf{Cof}, \mathsf{Fib})$ be a model category. Let $A \in \operatorname{Ob} \mathsf{M}$. Show that there are model structures on M/A and A/M with respect to which the forgetful functors $\mathsf{M}/A \to \mathsf{M}$ and $A/\mathsf{M} \to \mathsf{M}$ preserve the three distinguished classes of morphisms.

2. Let C be a bicomplete category, and let S be a class of maps in C. For any non-empty ordinal λ, seen as ordered set and thus also as a category, and any functor $F : \lambda \to \mathsf{C}$ such that $F(\alpha) \to F(\alpha + 1)$ is in S for every $\alpha + 1 < \lambda$, the canonical map $F(0) \to \operatorname{colim}_\lambda F$ is called a *transfinite composite of morphisms in S*.

   (a) Show that $\mathrm{LLP}(S)$ is closed under coproducts and under transfinite composites.

   (b) Formulate and prove the dual statement for $\mathrm{RLP}(S)$.

3. Let $(\mathsf{M}, \mathsf{WE}_1, \mathsf{Cof}_1, \mathsf{Fib}_1)$ and $(\mathsf{M}, \mathsf{WE}_2, \mathsf{Cof}_2, \mathsf{Fib}_2)$ be model categories such that $\mathsf{WE}_1 \subseteq \mathsf{WE}_2$ and $\mathsf{Fib}_1 \subseteq \mathsf{Fib}_2$. Prove that there is a model structure $(\mathsf{M}, \mathsf{WE}, \mathsf{Cof}, \mathsf{Fib})$ with $\mathsf{WE} = \mathsf{WE}_2$ and $\mathsf{Fib} = \mathsf{Fib}_1$.

4. Find three distinct model structures on the category Set.

5. Prove that there is a model structure on Cat for which the weak equivalences are equivalences of categories and the cofibrations are functors that are injective on objects.