Exercise 3 is to be handed in on 26.02.2018.

1. Let $X$ be a topological space. Prove that the following maps are continuous.
   
   (a) $\eta_X: X \to P(X \times I): x \mapsto (t \mapsto (x, t))$
   
   (b) $\text{ev}_X: PX \times I \to X: (\lambda, t) \mapsto \lambda(t)$
   
   (c) $\gamma: X \to PX: x \mapsto c_x$, where $c_x$ denotes the constant path at $x$.

2. A continuous map $f: X \to Y$ is said to be a retract of a continuous map $g: W \to Z$ if there are continuous maps $i: X \to W$, $r: W \to X$, $j: Y \to Z$, and $s: Z \to Y$ such that $ri = \text{Id}_X$, $sj = \text{Id}_Y$, and the diagram

   $\begin{array}{ccc}
   X & \xrightarrow{i} & W \xrightarrow{r} X \\
   f \downarrow & & \downarrow f \\
   Y & \xrightarrow{j} & Z \xrightarrow{s} Y \\
   \end{array}$

   commutes.

   Suppose that $f$ is a retract of $g$. Prove that if $g$ is a homotopy equivalence, then so is $f$.

3. For any map $f: X \to Y$, let $PX = \{(x, \mu) \in X \times PY \mid f(x) = \mu(0)\}$, equipped with the subspace topology of the product $X \times PY$.

   (a) Show that the map $q_f: PX \to P: \lambda \mapsto (\lambda(0), f \circ \lambda)$ is continuous.

   (b) Prove that a continuous map $p: E \to B$ is a Hurewicz fibration if and only if $q_p$ admits a section, i.e., there is a continuous $s: P_E \to PE$ such that $q_p \circ s = \text{Id}_{P_E}$.

   (c) Use this characterization to prove that

   $\left(ev_0, ev_1\right): PX \to X \times X: \lambda \mapsto (\lambda(0), \lambda(1))$

   is a Hurewicz fibration.