1. Let $A$ be a subspace of $X$, and let $x_0 \in A$. Let $P_i$ denote the homotopy fiber of the inclusion $i : A \hookrightarrow X$ (cf. Definition 2, Exercise Set 8). Show that

$$\pi_n(X, A) \cong \pi_{n-1}P_i.$$ 

2. Let $\{x_0\} \subseteq A \subseteq B \subseteq X$ be a sequence of subspace inclusions. Show that there is an exact sequence

$$\cdots \rightarrow \pi_n(B, A) \rightarrow \pi_n(X, A) \rightarrow \pi_n(X, B) \xrightarrow{\partial} \pi_{n-1}(B, A) \rightarrow \cdots.$$ 

**Hint:** The connecting homomorphism $\partial_n$ is equal to the composite

$$\pi_n(X, B) \rightarrow \pi_{n-1}B \rightarrow \pi_{n-1}(B, A).$$ 

3. Let $j : A \hookrightarrow X$ denote the inclusion of a closed subspace.

   (a) Prove that if $j$ is a cofibration, then so is

   $$j \times Id_Z : A \times Z \rightarrow X \times Z$$

   for all spaces $Z$.

   (b) Prove that if $j$ is a cofibration, then for all maps $f : A \rightarrow Y$, the induced inclusion

   $$Y \hookrightarrow Y \cup_f X = Y \coprod X/ \sim,$$

   where $a \sim f(a)$ for all $a \in A$, is also a cofibration, i.e., cofibrations are preserved under pushout. In particular, for all maps $f : X \rightarrow Y$, the inclusion $i_f : Y \hookrightarrow C_f$ is a cofibration.

4. Prove that if $i : A \hookrightarrow B$ and $j : B \hookrightarrow X$ are cofibrations, then so is $j \circ i : A \hookrightarrow X$. Prove more generally that if

   $$A_0 \xrightarrow{i_0} A_1 \xrightarrow{i_1} A_2 \xrightarrow{i_2} \cdots$$
are inclusions of closed subspaces that are cofibrations, then the inclusion $A_0 \hookrightarrow \bigcup_{n \geq 0} A_n$ is a cofibration, if the topology on $\bigcup_{n \geq 0} A_n$ satisfies:

$$C \subseteq \bigcup_{n \geq 0} A_n \text{ closed } \iff C \cap A_n \text{ closed in } A_n \forall n.$$