1. Prove that the fundamental group of any $H$-space (not necessarily homotopy associative!) is abelian.

2. Let $f : (X, x_0) \to (X', x'_0)$ be a pointed, continuous map. Explain how to extend the based loop space and suspension constructions to define functors

$$\Omega : \textbf{Top}_* \to \textbf{H-spaces}$$

and

$$\Sigma : \textbf{Top}_* \to \textbf{co-H-spaces}.$$

3. For any pointed space $(X, x_0)$, define pointed, continuous maps

$$\iota_{(X,x_0)} : (X, x_0) \to \Omega \Sigma (X, x_0)$$

and

$$\rho_{(X,x_0)} : \Sigma \Omega (X, x_0) \to (X, x_0)$$

such that for all pointed, continuous maps $f : (X, x_0) \to (X', x'_0)$,

$$\Omega \Sigma f \circ \iota_{(X,x_0)} = \iota_{(X',x'_0)} \circ f : (X, x_0) \to \Omega \Sigma (X', x'_0),$$

and

$$f \circ \rho_{(X,x_0)} = \rho_{(X',x'_0)} \circ \Sigma f : \Sigma \Omega (X, x_0) \to (X', x'_0),$$

i.e., $\iota$ is a natural transformation from $\text{Id}_{\textbf{Top}_*}$ to $\Omega \Sigma$, while $\rho$ is a natural transformation from $\Sigma \Omega$ to $\text{Id}_{\textbf{Top}_*}$.

Moreover,

$$\alpha : \text{Map} (\Sigma (X, x_0), (X', x'_0)) \to \text{Map} ((X, x_0), \Omega (X', x'_0)) : g \mapsto \Omega g \circ \iota_{(X,x_0)}$$

and

$$\beta : \text{Map} ((X, x_0), \Omega (X', x'_0)) \to \text{Map} (\Sigma (X, x_0), (X', x'_0)) : g \mapsto \rho_{(X',x'_0)} \circ \Sigma g$$

are mutually inverse H-isomorphisms (i.e., homeomorphisms that are H-morphisms).