1. Complex projective space of dimension $n$ is

$$\mathbb{C}P^n = \mathbb{C}^{n+1} \setminus \{0\} / \sim$$

where $z \sim z'$ if and only if there exists $\lambda \in \mathbb{C}$ such that $z = \lambda z'$. Prove that $SP^n(S^2) \cong \mathbb{C}P^n$.

2. (Eilenberg-MacLane spaces)

**Definition 1.** Let $G$ be an abelian group, and let $n \geq 2$. An *Eilenberg-MacLane space of type* $(G,n)$ is a pointed space $(X,x_0)$ such that $\pi_n(X,x_0) \cong G$ and $\pi_k(X,x_0) = 0$ if $k \neq n$.

**Conventions 2.** An Eilenberg-MacLane space of type $(G,n)$ is usually denoted $K(G,n)$.

(a) Show that $SP(S^n)$ is an Eilenberg-MacLane space of type $(\mathbb{Z}, n)$ for all $n \geq 1$.

(b) Let $m \in \mathbb{N}$, and define $\lambda_m : S^1 \to S^1$ by $\lambda_m(e^{i\theta}) = e^{im\theta}$. Prove that $\pi_1 \lambda_m : \pi_1 S^1 \to \pi_1 S^1$ is given by multiplication by $m$.

(c) Prove that $SP(C\lambda_m)$ is an Eilenberg-MacLane space of type $(\mathbb{Z}/m,1)$.

**Hint 3.** Use the quasi-fibration obtained by applying $SP$ to the sequence $S^1 \xrightarrow{\lambda_m} S^1 \to C\lambda_m$.

(d) Let $n > 1$. Prove that $SP(\Sigma^{n-1}C\lambda_m)$ is an Eilenberg-MacLane space of type $(\mathbb{Z}/m, n)$.

(e) Let $G$ be any finitely generated abelian group, and let $n \geq 1$. Explain how to construct an Eilenberg-MacLane space of type $K(G,n)$.